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ME-401: Advanced Mechanical Vibrations
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Characterization of Closely-Spaced Mode Shapes of Polycarbonate Plate

Introduction

A plate with an aspect ratio of 1:1.66 is designed to exhibit pairs of closely-spaced modes. With closely-spaced modes, the plate can serve as a critical tool to validate the adequacy of data acquisition parameters. By observing whether these two modes are accurately captured, students can determine whether test setup and data acquisition parameters need adjustment. The experiment will perform modal analysis on both the original and the newly designed plates through shaker testing. Modal analysis is significant since it provides the vibrational characteristics of the plates, such as natural frequencies and mode shapes.

The objectives of this experiment are 1) to perform modal analysis on both an original and newly designed polycarbonate plate using a shaker, 2) to analyze and identify closely-spaced modes of the new plate, and 3) to compare these findings with the data obtained from the original plate.

Theory – Natural Frequencies of Rectangular Plates

The natural frequencies of a rectangular plate can be calculated using the following equation[1]:

$$f_{ij}[Hz] = \frac{\lambda_{ij}^2}{2\pi a^2}\left[\frac{Eh^2}{12\rho(1-\nu^2)}\right]^{1/2} ; i = 1,\ 2,\ 3 \ldots ; j = 1,\ 2,\ 3 \ldots$$

Equation 1. Equation for Plate Natural Frequencies

Where λ_{ij} is the dimensionless frequency parameter,

a is the length of the plate,

h is the thickness of the plate,

ρ is the density of the plate,

E is the Young's Modulus of the plate,

ν is the Poisson's ratio,

And i and j are the number of half waves in the mode shapes along the x- and y-coordinates, respectively.

λ_{ij} depends on the boundary conditions and aspect ratio of the rectangular plates. Considering the rectangular plate with the aspect ratio of 1:1.66 and all free edges, λ_{ij}^2, natural frequency parameters, can be found in Table 5.3 in *Formulas for Dynamics, Acoustics and Vibration*, which is also shown Table 1. Due to the limited available options, the aspect ratio of 1:1.5 was chosen.

Mode Sequence	1	2	3	4	5
$\boldsymbol{\lambda_{ij}^2}$ **(ij)**	20.13 (22)	21.60 (31)	46.65 (32)	50.29 (13)	50.20 (41)

Table 1. Natural Frequency Parameter when Aspect Ratio = 1:1.5

[1] (Blevins)

Equipment

Equipment	Specifications
Shaker	Modal Shop K2007E01
Force Transducer	PCB Y208C02
Accelerometer	PCB 333B31 / 333B53
Data Acquisition Tool	NI USB-4431

Table 2.List of Equipment

Experiment

Shown in Figure 1, the dimension of the original plate (Plate 1) is 12.60 inches by 20.87 inches with a thickness of 7/12 inches. As depicted on the left side of Figure 2, 24 equally spaced points were marked on the plate. Similarly, the newly designed plate, shown in Figure 2 and on the right side of Figure 3, measures 12 inches by 20 inches with a thickness of ½ inch. It also has 24 equally spaced points. The properties of the plates are detailed in Table 3. Plate 2 was manufactured by cutting a piece of polycarbonate sheet to the correct size on a vertical bandsaw. At the predetermined measurement and excitation points, a blind hole was drilled and then tapped to the correct dimensions for mounting a force transducer. This will allow future students to excite the plate from any of the 24 equally-spaced points.

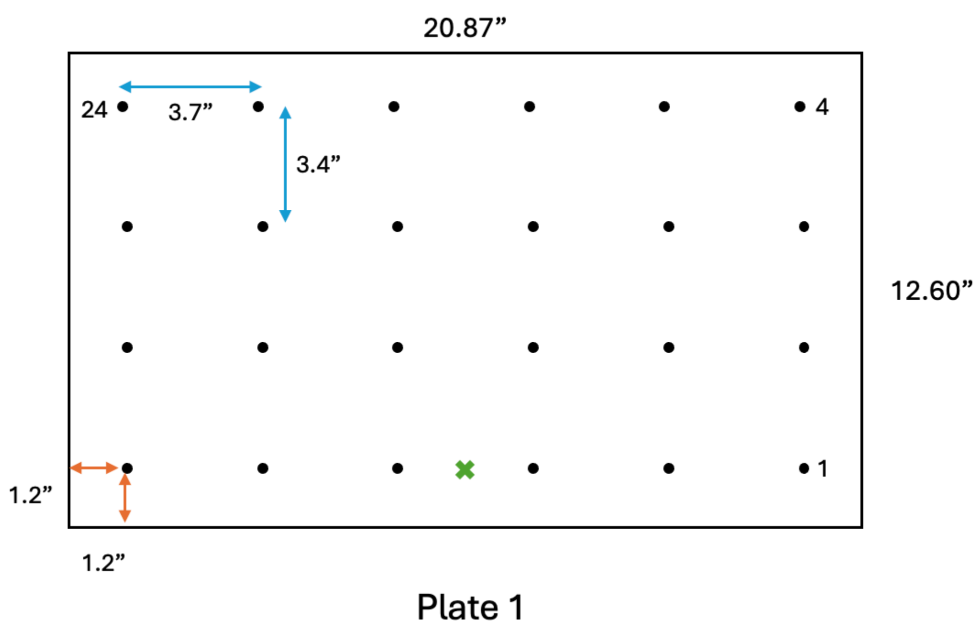


Figure 1. Dimensions of Plate 1 (Original)

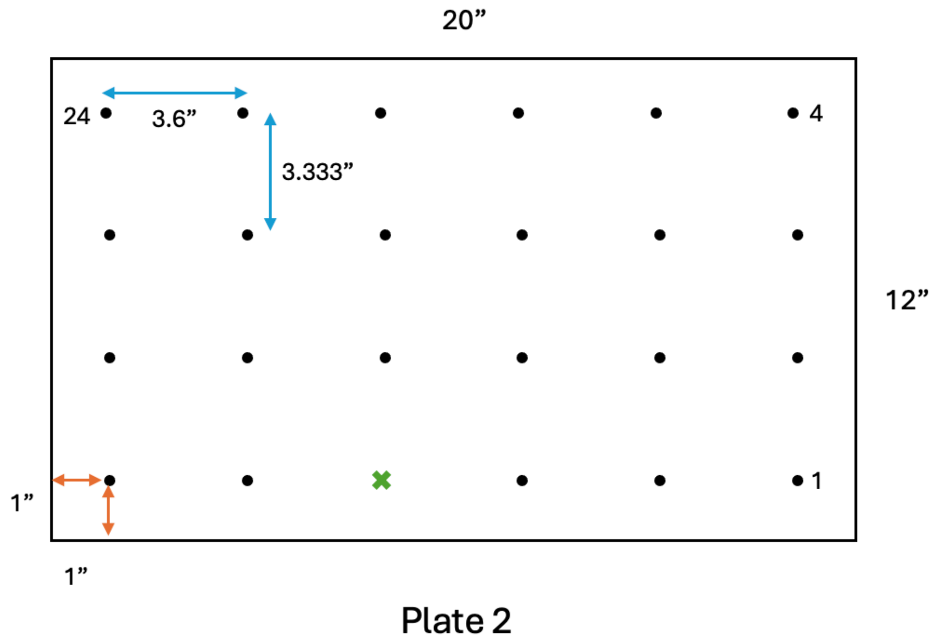


Figure 2. Dimensions of Plate 2 (New)

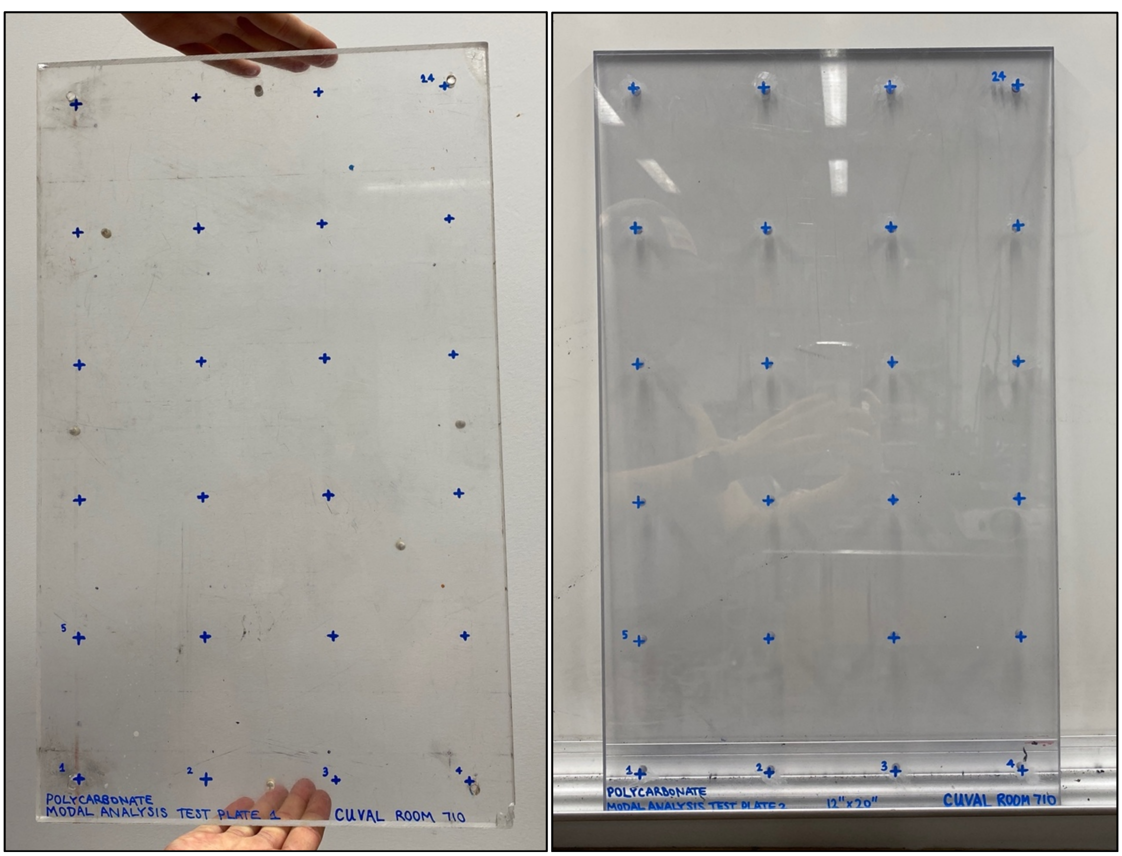


Figure 3. Original Plate, or Plate 1, (left) and Newly-designed Plate, or Plate 2 (right)

	Plate 1	Plate 2
Material	Polycarbonate	
Density (ρ)*	0.043 lbm/in^3	
Young's Modulus (E)**	1.958e6 psi	
Poisson's Ratio (ν)*	0.318	
Length (a)	20.87"	20"
Width (b)	12.60"	12"
Thickness (h)	7/12"	½"

Table 3. *Plate Properties and Dimensions*

Notes: Source – * AZO Materials; ** McMaster-Carr

Figure 4 illustrates how the plate was supported by cross-tied latex rubber bands, which was the best way we could think to approximate a free boundary condition on all four edges of the plate. The green cross mark shown in Figures 1 and 2 indicates the location where the force transducer was attached. The force transducer was positioned midway between Points 9 and 13 on Plate 1, and directly at Point 13 on Plate 2.

During the data collection process, the response of the plate was captured in sets of three data points. Three accelerometers were attached to the plate at a time during each shaker test. The testing was conducted repeatedly until data from all 24 points and the driving measurement point were collected. The data acquisition parameters for the experiment are detailed in Table 4.

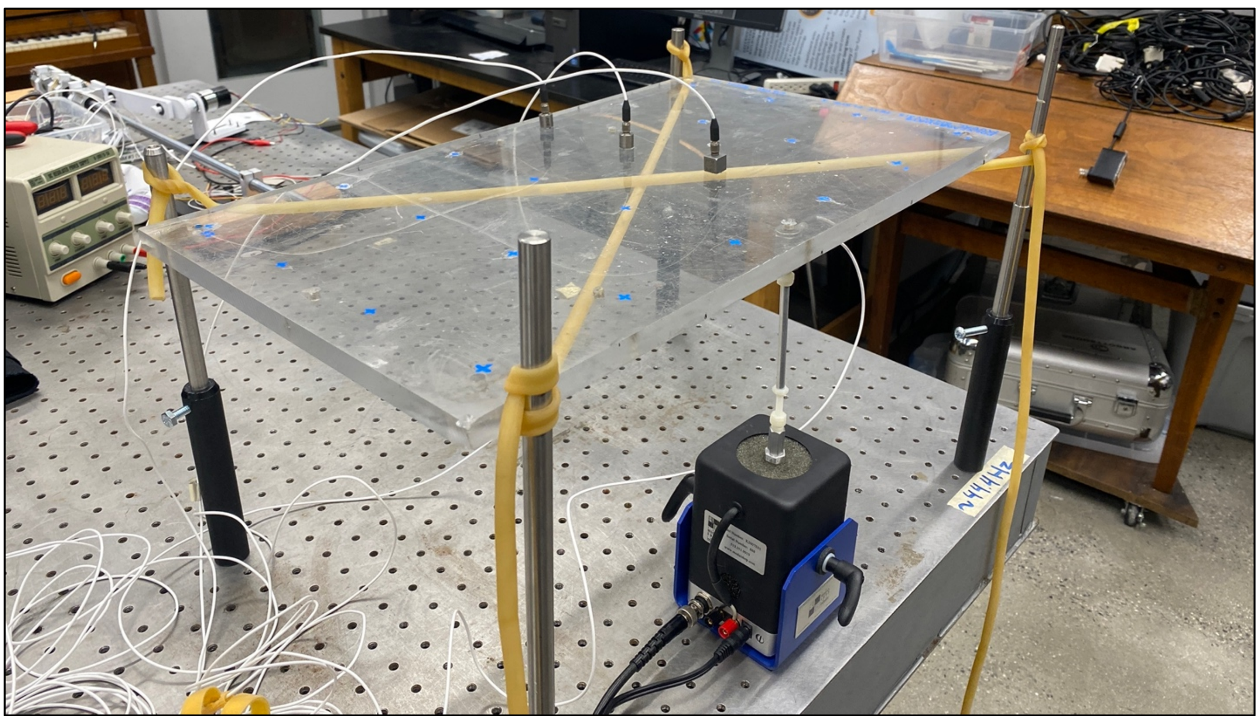


Figure 4. *Test Setup*

Parameter		**Value**
Input Signal	Acquisition mode	N Samples
	Samples to read	40,000
	Rate	2,000 S/s
Output Signal	Signal type	Sine Sweep (Formula – Repeated signal)
	Frequency	2.5 Hz
	Amplitude	200 mV
	Formula	A*sin((12.5*t*w)*t)
FRF Configuration	Window	Hanning
	Number of averages	2

Table 4. Data Acquisition Parameters

Results

FRF – Magnitude Plots

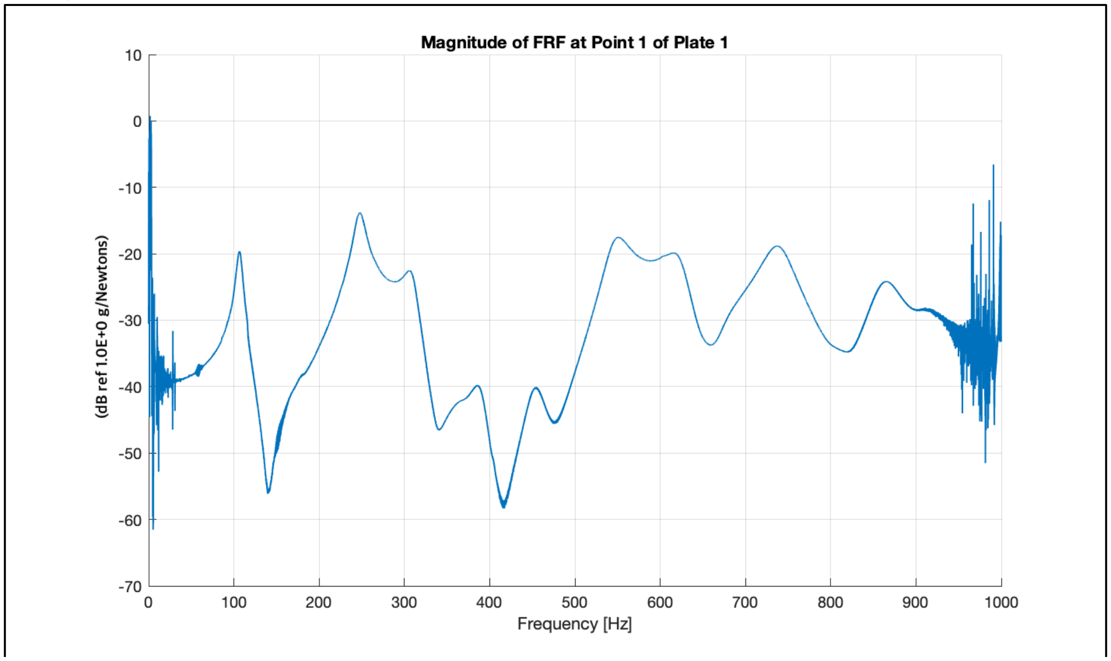


Figure 5. Magnitude of FRF at Point 1 of Plate 1

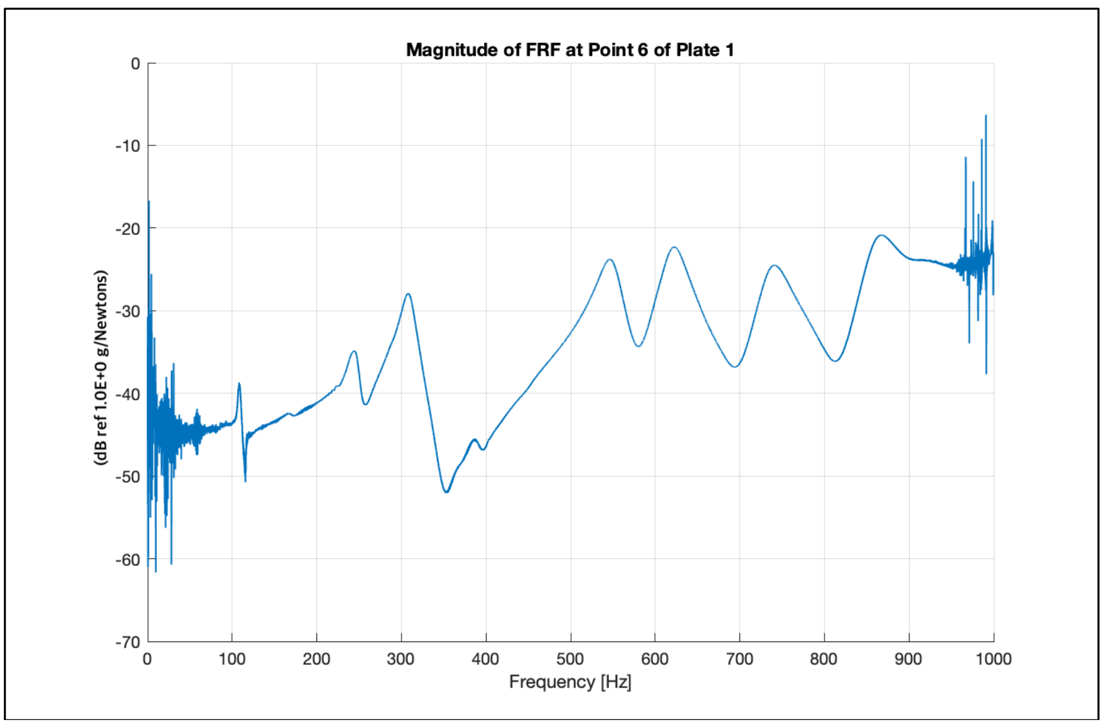


Figure 6. Magnitude of FRF at Point 6 of Plate 1

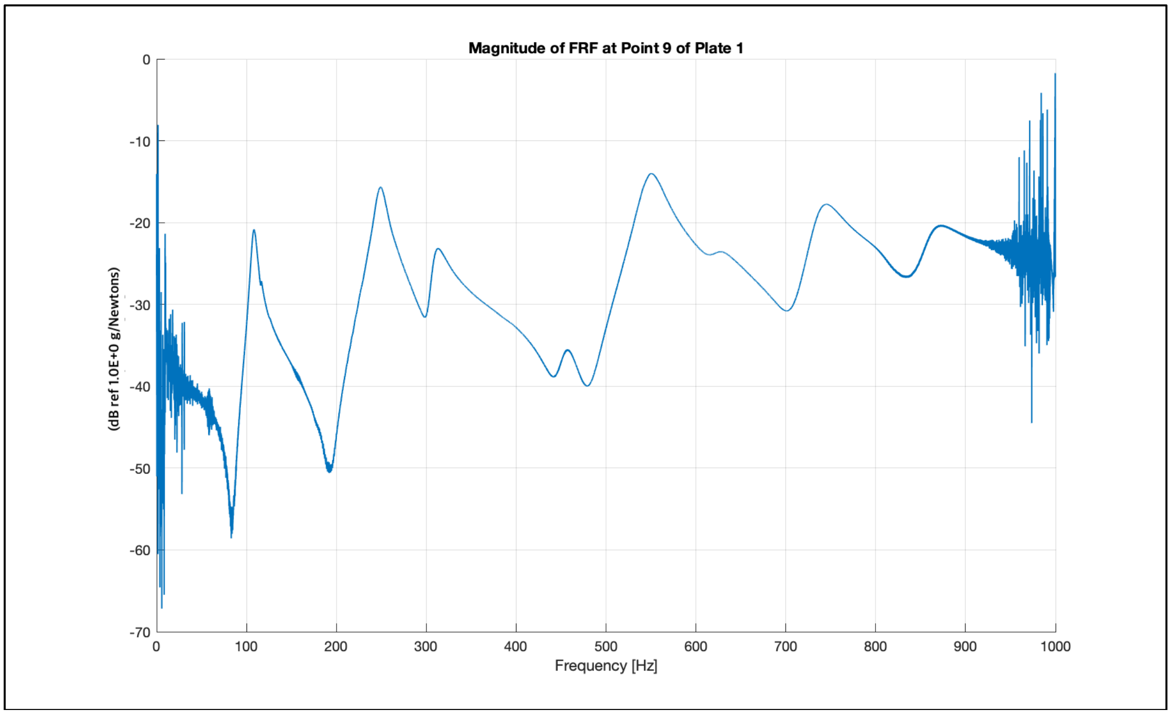


Figure 7. Magnitude of FRF at Point 9 of Plate 1

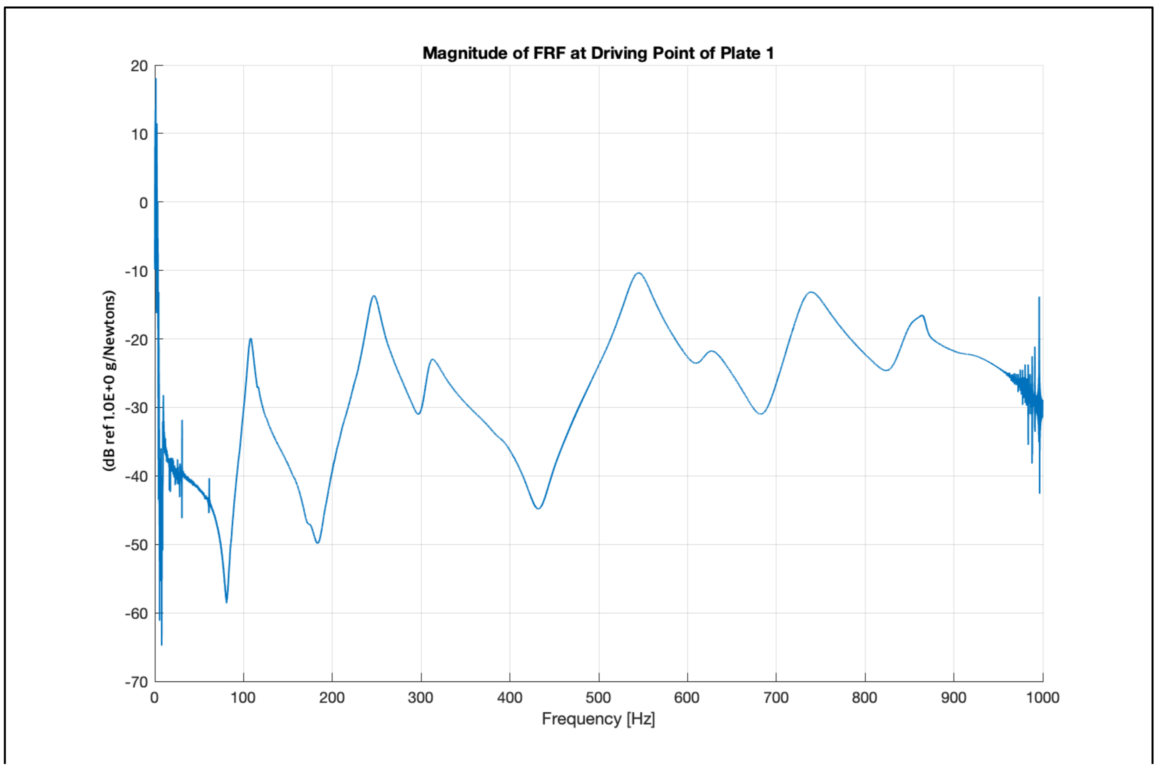


Figure 8. Magnitude of FRF at Driving Point of Plate 1

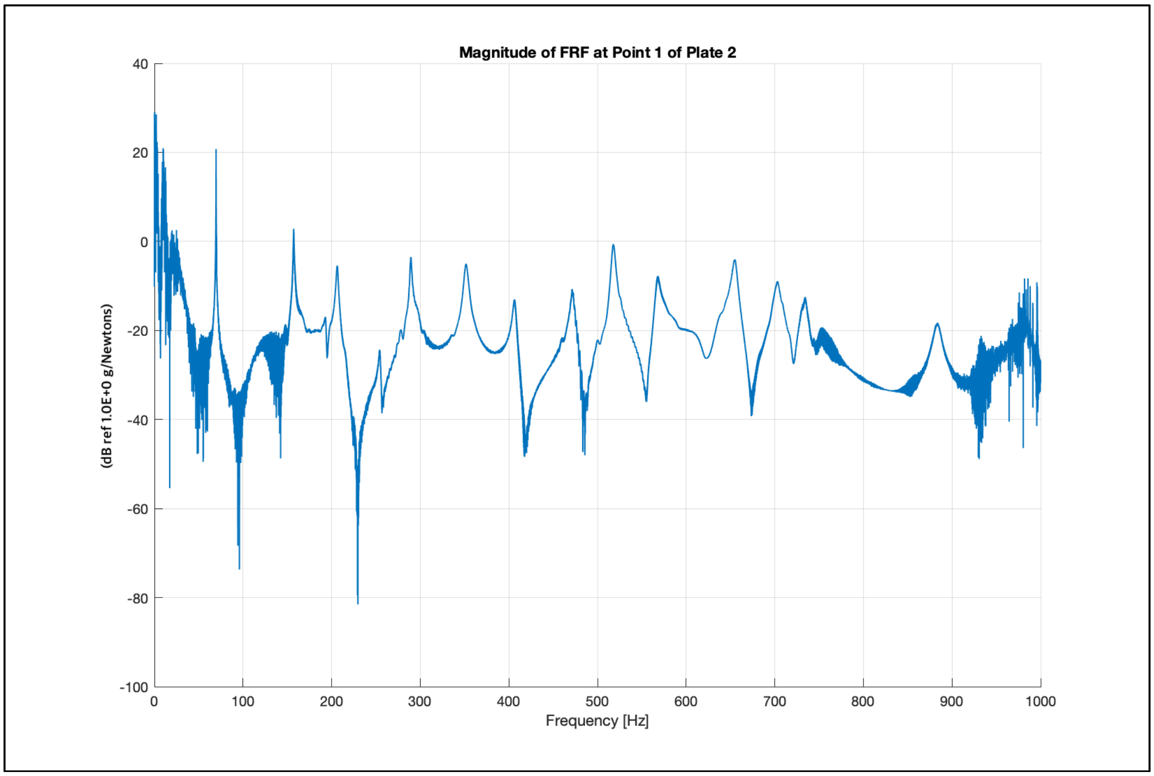


Figure 9. Magnitude of FRF at Point 1 of Plate 2

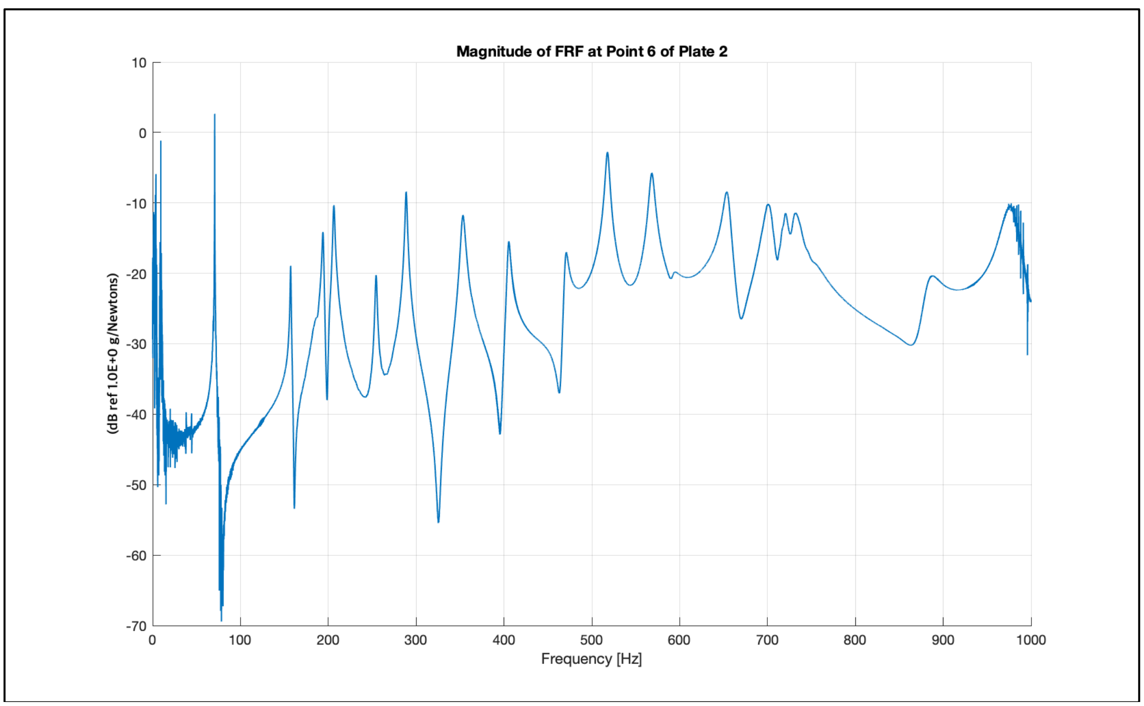


Figure 10. Magnitude of FRF at Point 6 of Plate 2

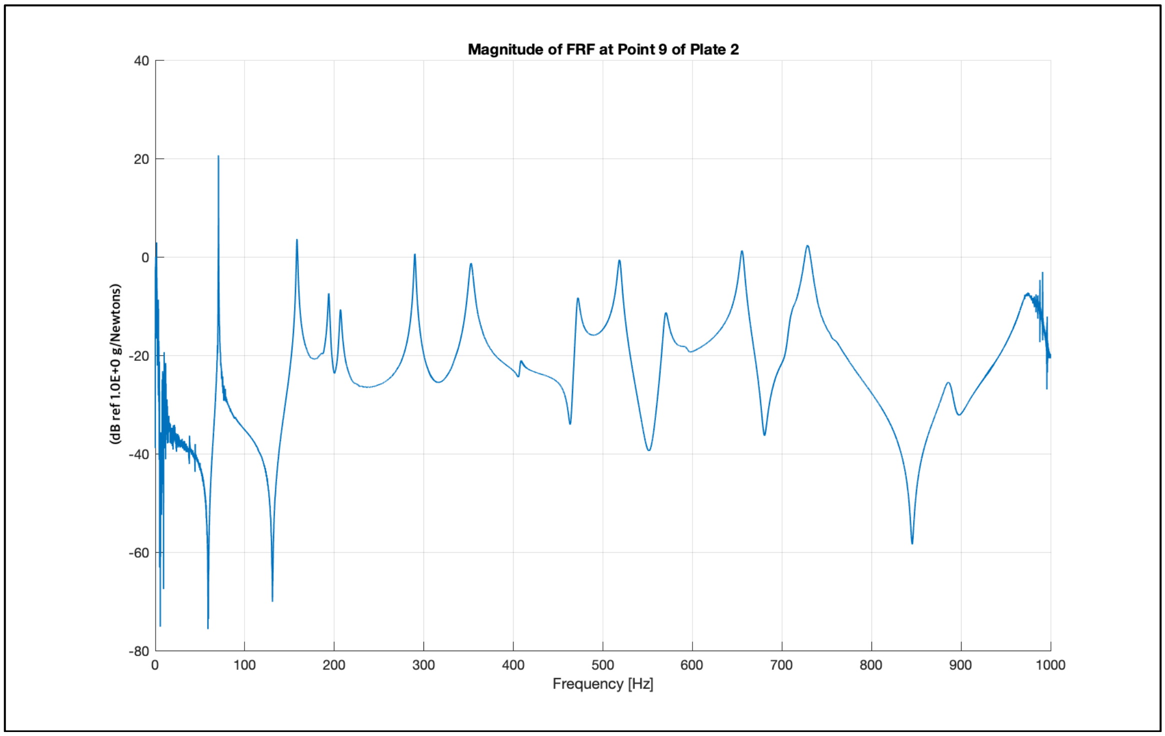


Figure 11. Magnitude of FRF at Point 9 of Plate 2

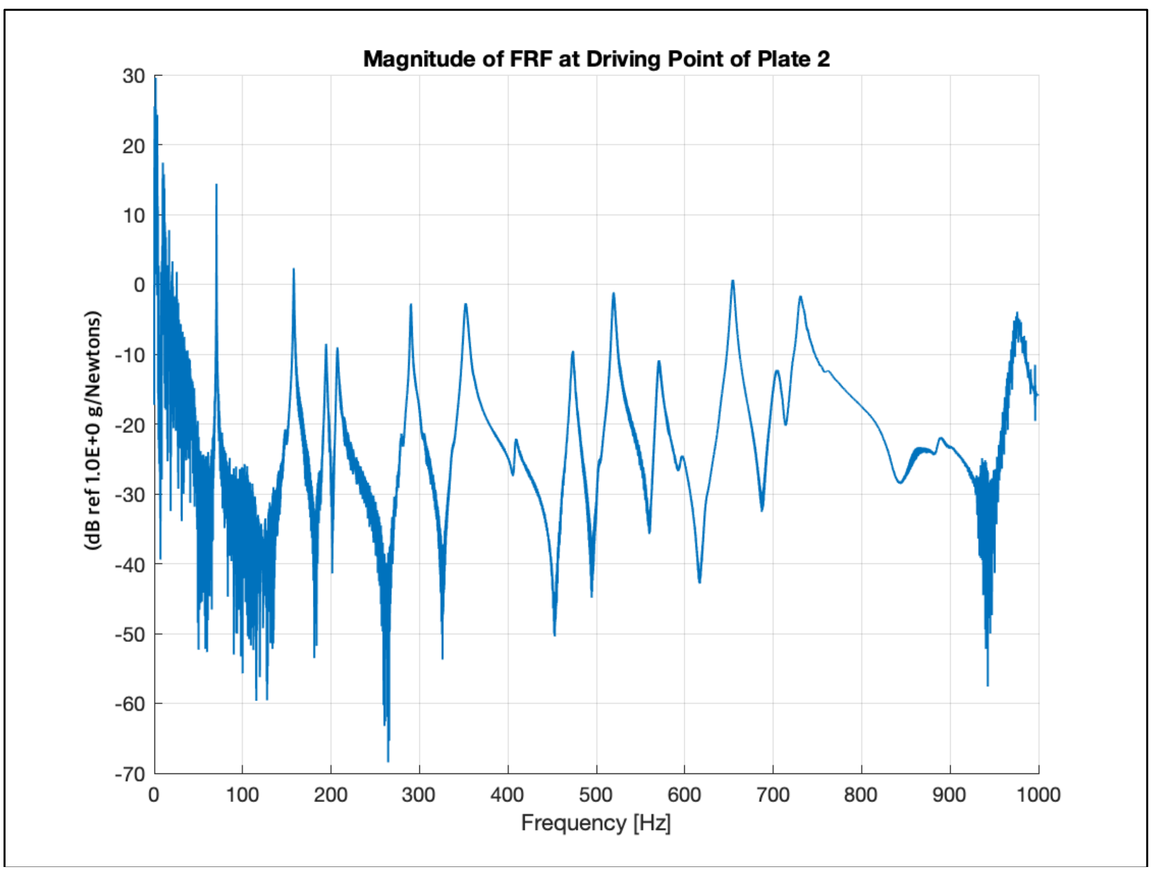
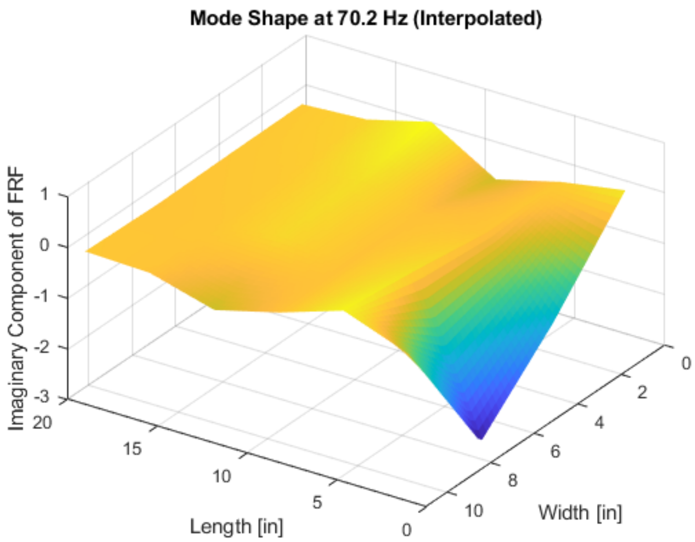
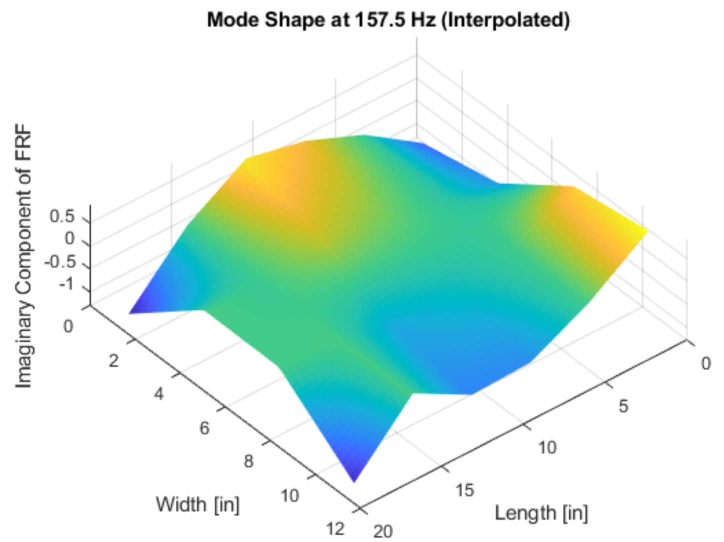
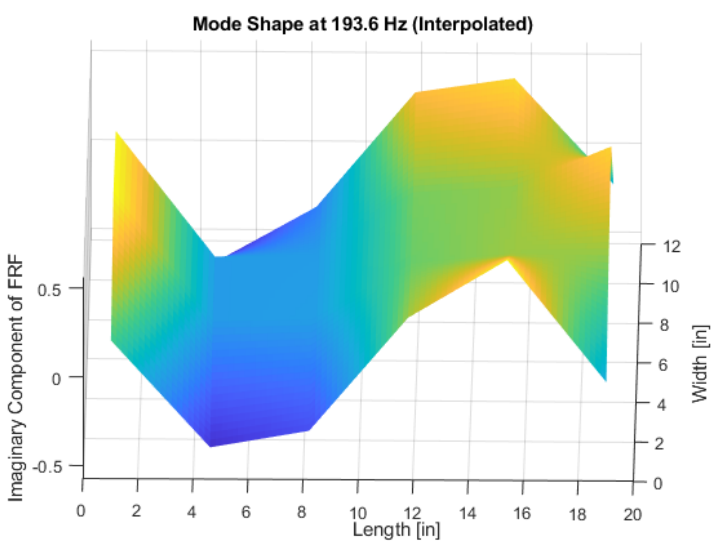


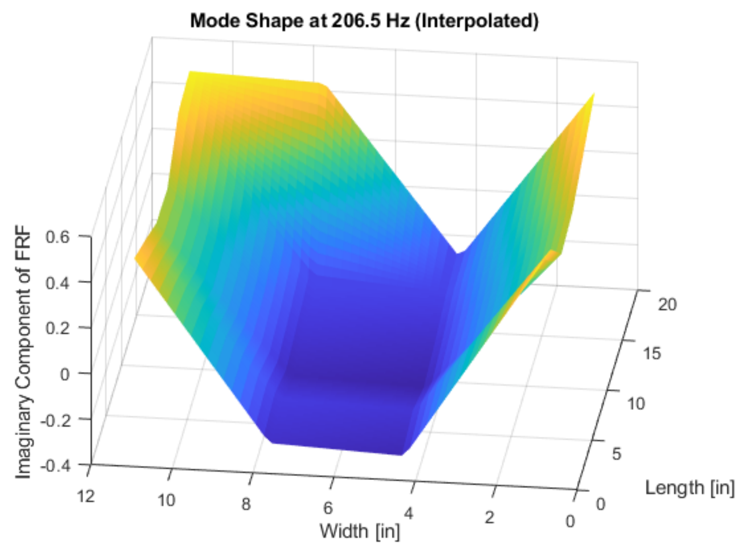
Figure 12. Magnitude of FRF at Driving Point of Plate 2

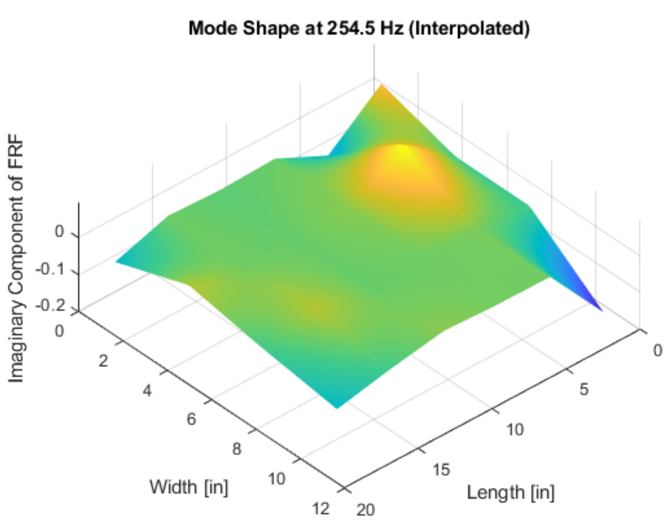
Plate 2 Experimental Modeshapes

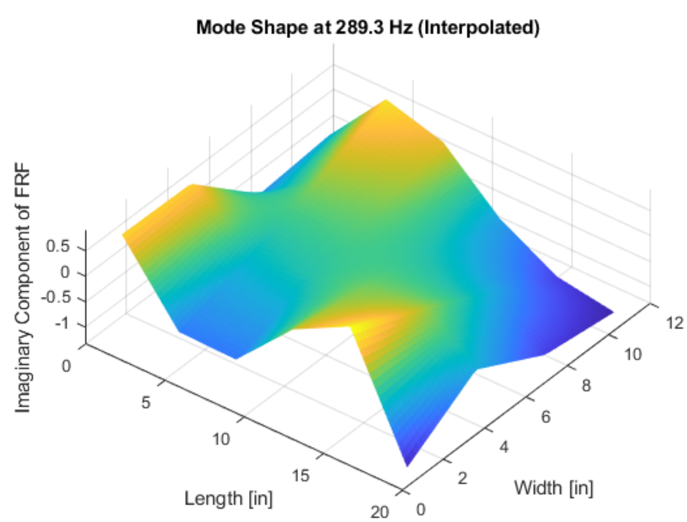












Discussion

The theoretical natural frequencies in Table 5 were derived using Equation 1.

	Theoretical Natural Frequency [Hz]	
ij	**Plate 1**	**Plate 2**
22	173.2	161.7
31	185.9	173.5
32	401.4	374.7
13	432.8	403.9
41	432.0	403.2

Table 5. *Theoretical Natural Frequency of Plates 1 and 2*

These natural frequencies are notably different from those determined experimentally, shown in Tables 6 and 7.

Plate 1: Frequency of Peak [Hz]			
Point 1	**Point 6**	**Point 9**	**Driving Point**
106.65	108.10	108.55	107.95
247.70	244.55	249.35	247.05
306.40	308.05	313.65	313.00
385.15	-	-	-
454.50	-	-	-
551.25	546.50	550.50	-
615.40	622.45	-	626.95
737.30	741.05	744.95	739.30
865.35	865.80	872.20	863.95

Table 6. *Frequency of Peak on Points 1,6, 9 and Driving Point of Plate 1*

Plate 2: Frequency of Peak [Hz]			
Point 1	**Point 6**	**Point 9**	**Driving Point**
69.60	70.70	70.60	70.40
157.15	157.15	158.20	157.75
192.90	193.85	193.80	194.40
206.25	206.40	206.90	207.10
-	254.40	258.85	-
289.40	288.70	289.90	290.40
351.70	353.45	352.75	352.40
406.45	405.45	408.6	-
471.45	471.10	472.10	473.70
517.85	517.85	518.50	519.45
568.10	568.40	570.40	570.70
654.75	653.55	655.40	654.40
703.15	720.60	-	-
734.45	731.60	728.45	730.80

Table 7. *Frequency of Peak on Points 1,6, 9 and Driving Point of Plate 2*

This is likely caused by a few different factors. Most importantly, the natural frequency values were calculated for a rectangle with a side length ratio of 1:1.5 due to the limited ratio options available in the source used for the calculation. However, even if the theoretical frequency values were recalculated, they would likely still differ from the measured values. The other factor at play is the boundary condition setup for the experiment. The diagonal cross-shaped boundary condition used in this experiment is an improvement over previous test setups as it does not have a rubber tube directly supporting the ends of the plate. This ensures that the ends of the plate are actually free to move without being immediately resisted by the stiffness of the rubber tubing. It also distributes the load more evenly across the surface of the plate. However, the weight could be distributed further using a thin rubber sheet to support the entire bottom surface of the plate.

Plate 2 is an improvement over plate 1 because it is slightly smaller, thinner, and has a true 5:3 side length ratio. This allows the natural frequencies to become tightly spaced within the 1000 Hz range which is convenient for reducing experiment time and data storage, specifically between 193 Hz and 207 Hz as shown in Table 7, as well not running the shaker at higher frequencies where it may not respond as linearly.

The mode shapes for the natural frequencies are an interpolated plot of all the imaginary components at the measurement points at a specific frequency. These should be understandably questioned due to the fact that from measurement to measurement, the frequency position of the peaks shifts slightly with experimental error. This means that the imaginary component which corresponds to a certain mode of vibration may not have been queried correctly for every point in the plane. The peaks shifting is likely due to the position of the accelerometers moving after every three data points. Although the accelerometer masses are very small compared to that of the polycarbonate plate, they have the effect of shifting the natural frequencies of the plate by a few hertz. The method could have been improved by using a modal stability diagram to identify the natural frequencies for each data point before querying the imaginary component instead of applying the same natural frequencies to every data point. Unfortunately, this process was limited by the mistake of not downloading the real component of the FRF which is necessary for the modal stability algorithm.

Despite the inaccuracy inherent in the mode shape plots, three notable plots are the mode shapes at 193.6, 206.5, and 289.3 Hz. The first has a sinusoidal two-half wavelength shape along the length which seems to correspond an ij = 21 mode shape. The second seems like it could be an ij = 22 mode shape with each side containing one whole wavelength. The third looks very similar to an ij = 32 mode shape with three-half wavelengths along the long end of the plate and an inversion in the magnitude along the width of the plate. The fact that these correspond in exactly the order in which is a very satisfying result.

Conclusion

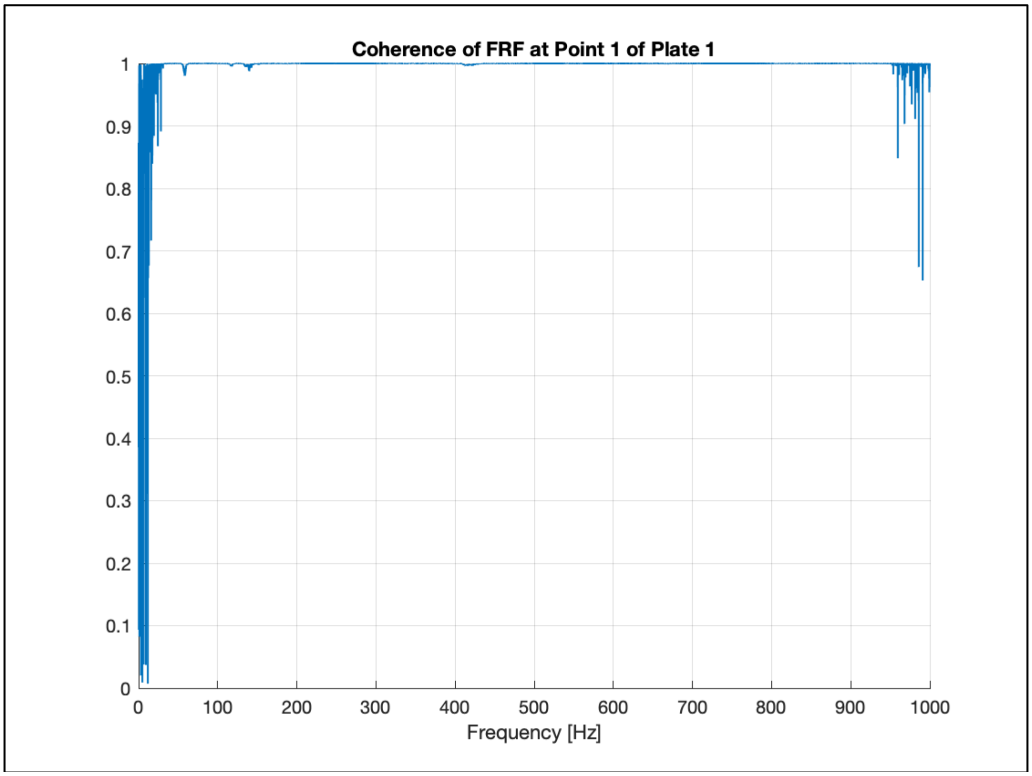
One column of a 24x24 FRF matrix was determined experimentally for both Plate 1 and Plate 2 with the goal of understanding MIMO and continuous systems. The experimental natural frequencies of both plates were examined and compared to theoretical values. Mode shapes for the plates were plotted and process improvements were determined for future data collection. Importantly, it is necessary to save all data from the FRF especially both real and imaginary parts. Future work can involve using improved boundary conditions as described previously. A thin rubber sheet which spans the length or width of the plate could possibly be a better approximation of the free-free-free-free boundary condition while keeping the shaker set up the same. It is also recommended to test the polycarbonate plate with different boundary conditions and see if two closely-spaced modes are still observed. If time permits, perhaps rove the accelerometer one at a time to limit changes in the system between each data collection cycle. Alternatively, use a modal stability diagram or other automated natural frequency identification techniques to limit the amount of work required to achieve an accurate mode shape plot.

References

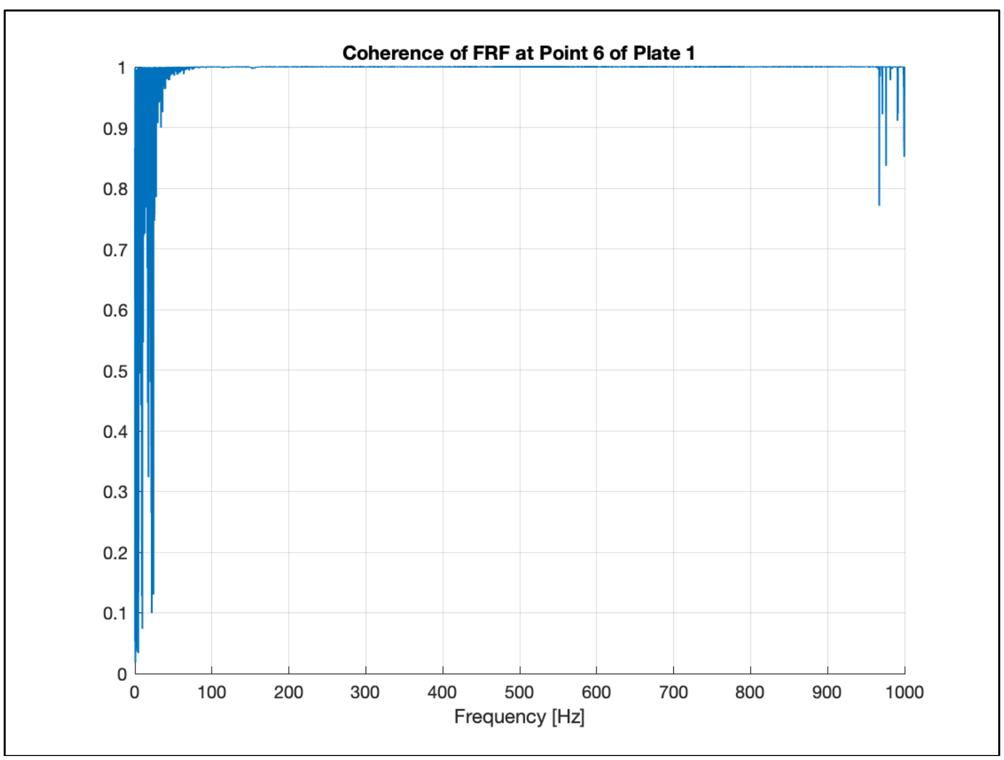
Baglione, Melody. “ME401_9_Impact_Testing_S10.”

Blevins, Robert D. Formulas for Dynamics, Acoustics and Vibration. John Wiley & Sons, 3 May 2016.

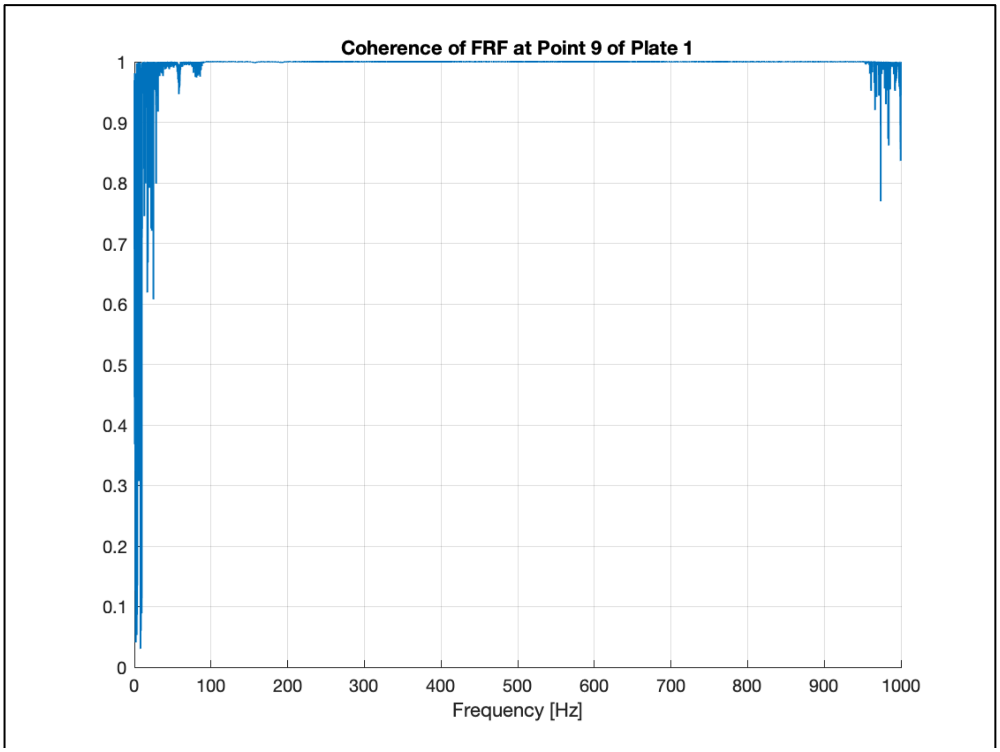
Appendix A: Coherence



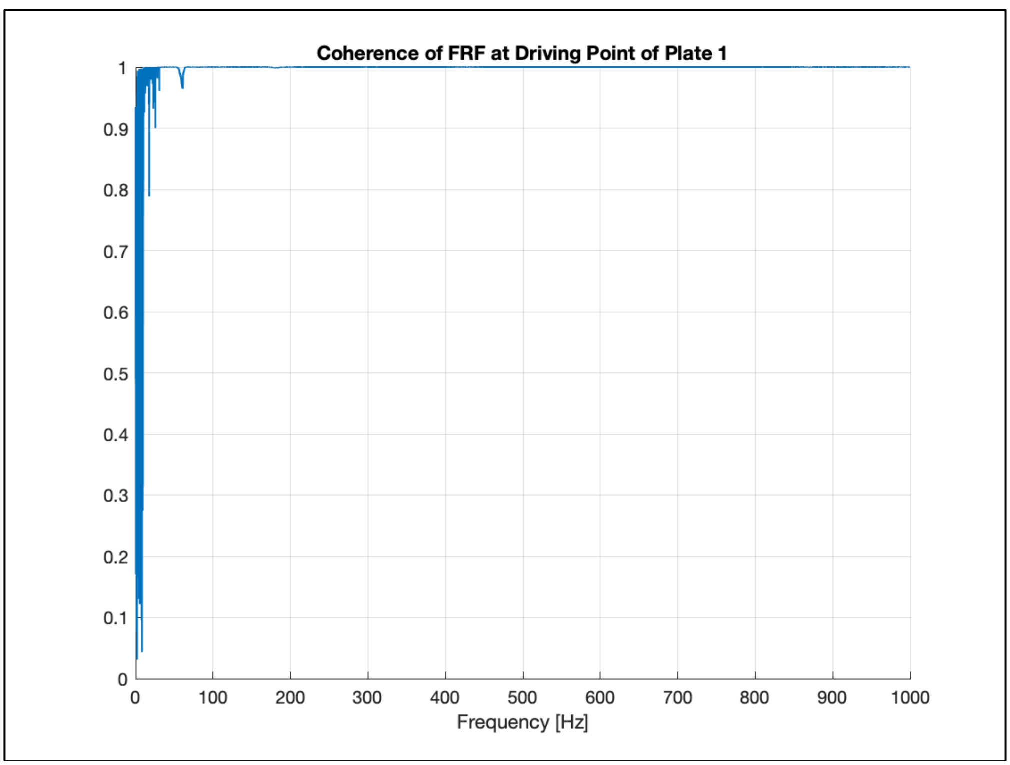
Coherence at Point 1 of Plate 1



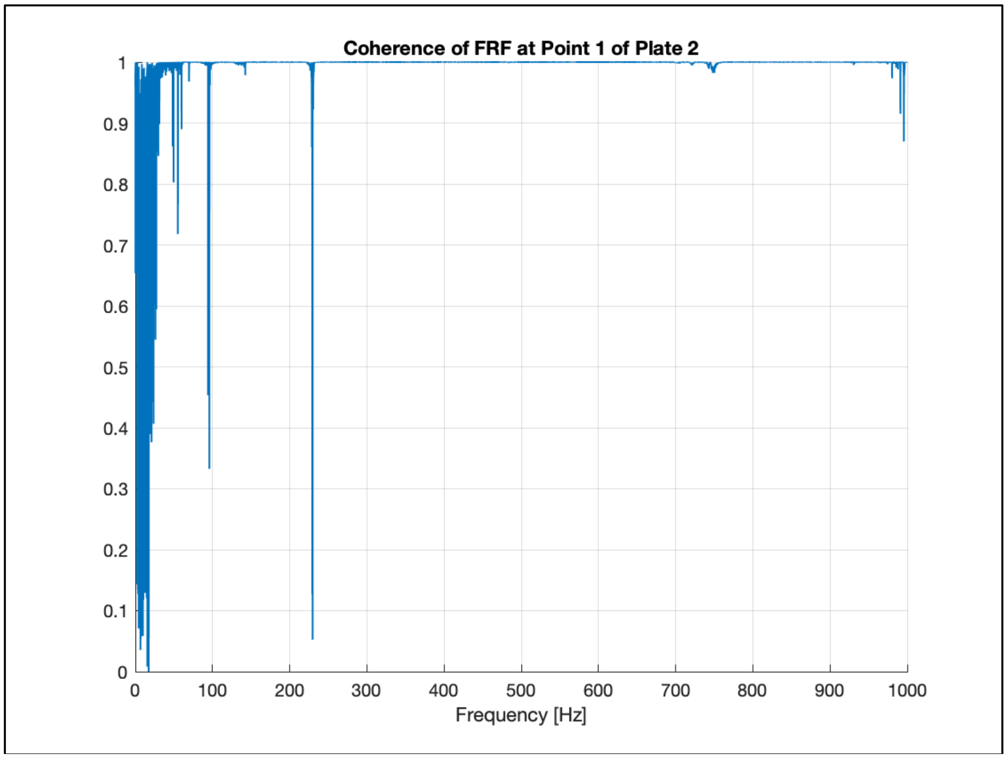
Coherence at Point 6 of Plate 1



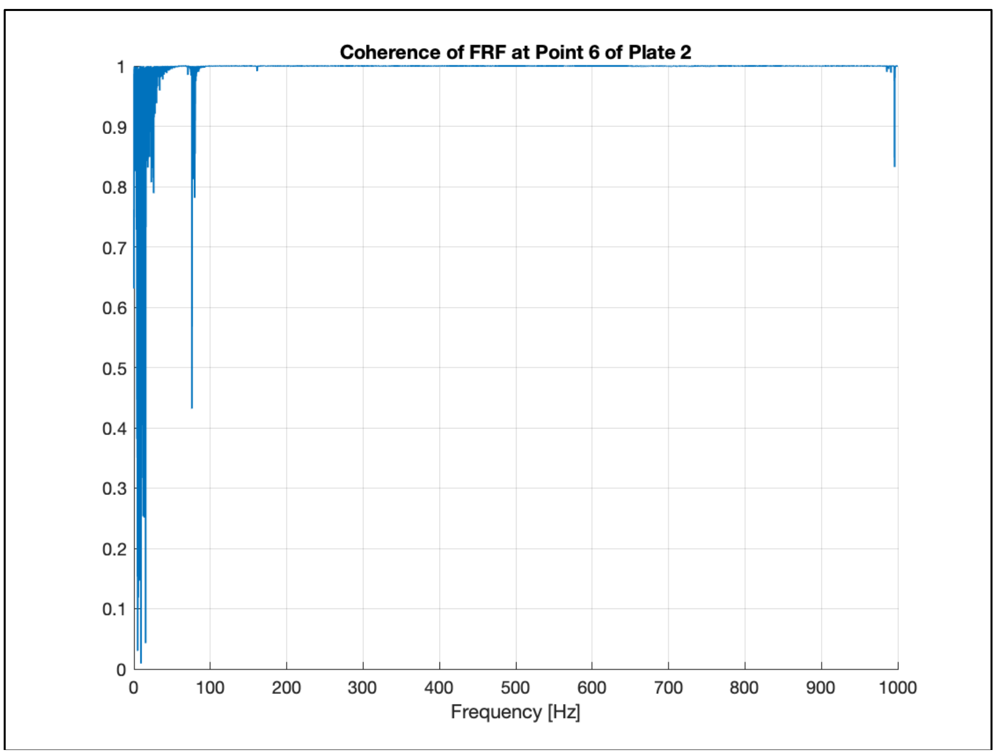
Coherence at Point 9 of Plate 1



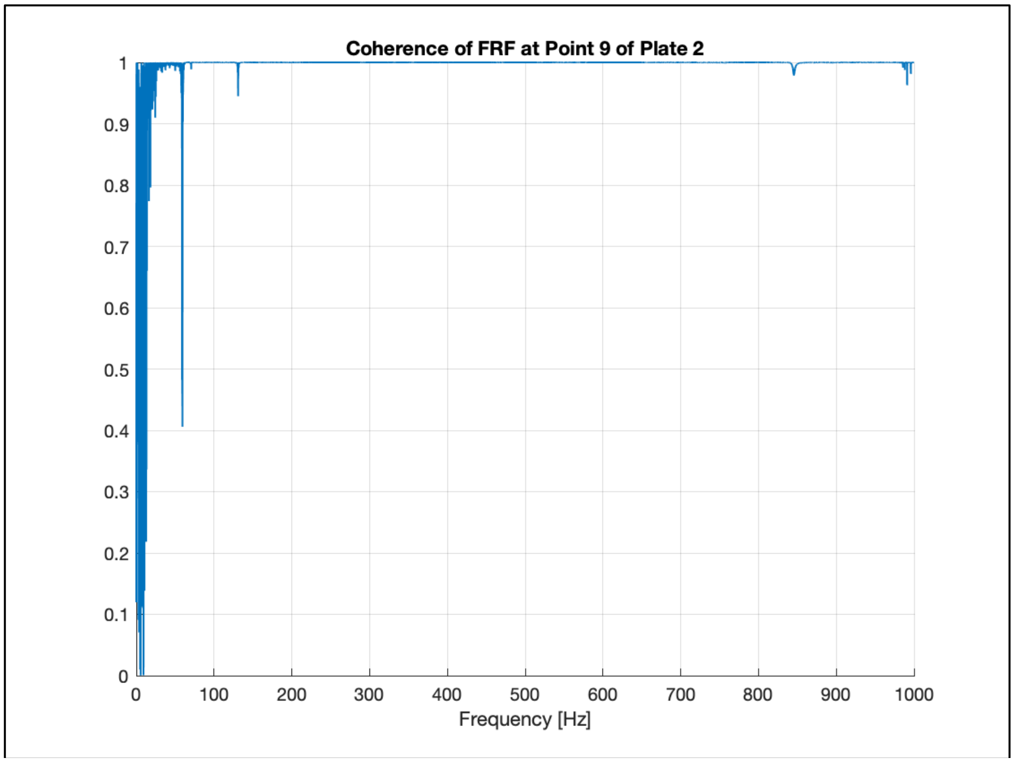
Coherence at Driving Point of Plate 1



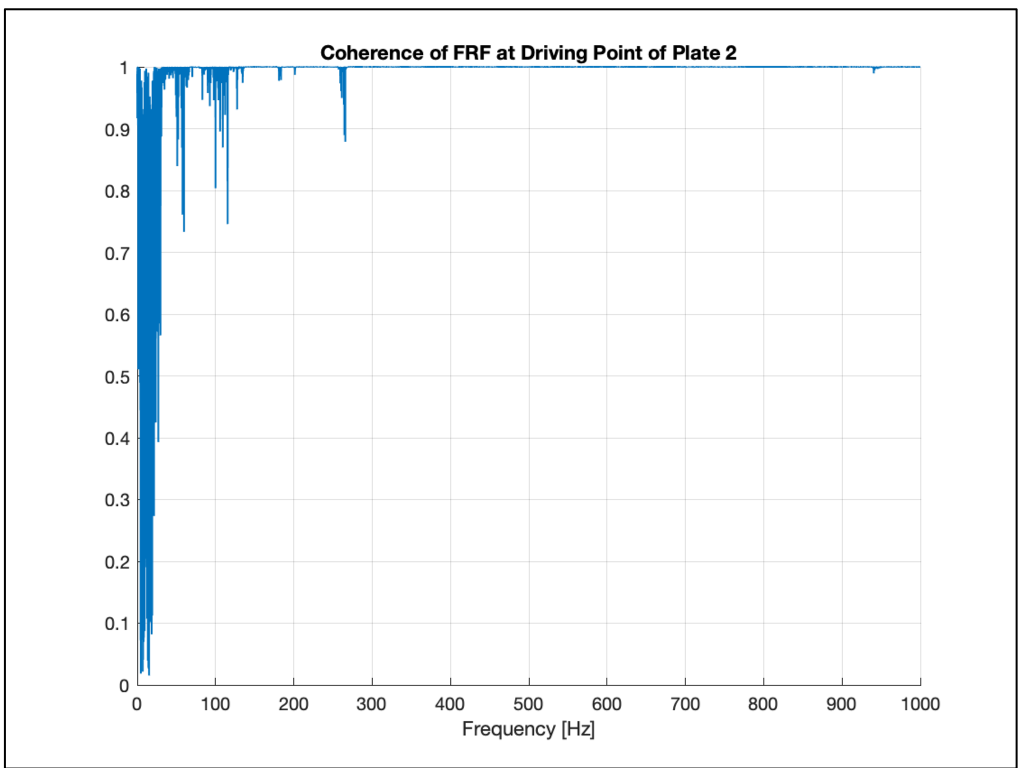
Coherence at Point 1 of Plate 2



Coherence at Point 6 of Plate 2



Coherence at Point 9 of Plate 2



Coherence at Driving Point of Plate 2

Appendix B: MATLAB code for plotting mode shapes

```
% Script written by Calder Leppitsch
% 5/9/2024
% Plotting modes of Plate 2

clc
clear
close all

% Loading data
im_sheetnames = {'imaginary resp...E) - Dev6_a(3)','imaginary resp...E) - 
Dev6_a(2)','imaginary resp...E) - Dev6_ai0)'};
mag_sheetnames = {'magnitude resp...E) - Dev6_a(3)','magnitude resp...E) - 
Dev6_a(2)','magnitude resp...E) - Dev6_ai0)'};
datalocation = 'C:\Users\C\Downloads\plate2';
datafilenames = 
{'1to3.xlsx','4to6.xlsx','7to9.xlsx','10to12.xlsx','13to15.xlsx','16to18.xlsx','19to2
1.xlsx','22to24.xlsx'};
n_files = length(datafilenames);

% Natural frequencies identified by Sarah
naturalfrequencies = 
[70.2;157.5;193.6;206.5;254.5;289.3;352.5;406.7;471.1;518;568.6;655.1;711.7;728.5];
n_nf = length(naturalfrequencies);
n_points = 24;

[Magnitude, Frequency, Imaginary] = deal(cell(n_points,1)); % Allocate Memory
% Turn this on to load the data for the first time from excel files. This
% takes a while so it is prefereable to save the data as a .mat file once
% it has been loaded.
spinagain = 0;
if spinagain == 1
    for point = 1:n_points
        filenumber = floor((point-1)/3)+1;
        filename = datafilenames{filenumber};
        sheetnumber = mod((point-1),3)+1;
        im_sheetname = im_sheetnames{sheetnumber};
        im_opts = spreadsheetImportOptions(NumVariables = 2,Sheet=im_sheetname);
        im_loadcell = readmatrix(strcat(datalocation,'\',filename),im_opts);
        resolution = length(im_loadcell)-9;
        Frequency_temp = zeros(resolution,1);
        Imaginary_temp = zeros(resolution,1);
        for i=1:resolution
            Frequency_temp(i) = str2double(im_loadcell{9+i,1});
            Imaginary_temp(i) = str2double(im_loadcell{9+i,2});
        end
        Frequency{point} = Frequency_temp;
        Imaginary{point} = Imaginary_temp;

        mag_sheetname = mag_sheetnames{sheetnumber};
        mag_opts = spreadsheetImportOptions(NumVariables = 2,Sheet=mag_sheetname);
        mag_loadcell = readmatrix(strcat(datalocation,'\',filename),mag_opts);
        resolution = length(mag_loadcell)-9;
        Magnitude_temp = zeros(resolution,1);
        for i=1:resolution
```

```
            Magnitude_temp(i) = str2double(mag_loadcell{9+i,2});
        end
        Magnitude{point} = Magnitude_temp;
    end
    save("modaldata_plate2.mat","Frequency","Imaginary","Magnitude")
else
    load("modaldata_plate2.mat")
end

frequencies = Frequency{1}; % All the frequency vectors are the same
x_spacing = linspace(1,19,6); % Comes from the plate dimensions
x_pts = length(x_spacing);
y_spacing = linspace(1,11,4);
y_pts = length(y_spacing);
modal_surfs = cell(n_nf,1);
for i = 1:n_nf % Do each natural frequency one at a time
    modal_surf_temp = zeros(y_pts,x_pts);
    for point = 1:n_points
        % Get the imaginary value at this point and frequency
        [~,freq_index] = min(abs(frequencies-naturalfrequencies(i)));
        Imaginary_temp = Imaginary{point};
        % Place the imaginary component values into their correct positions
        x_point = floor((point-1)/4)+1;
        y_point = mod((point-1),4)+1;
        modal_surf_temp(y_point,x_point) = Imaginary_temp(freq_index);
    end
    modal_surfs{i} = modal_surf_temp;
end

x_interp = (0:0.2:20); % Interpolation points
y_interp = (0:0.2:12);
[x_spacing_grid, y_spacing_grid] = meshgrid(x_spacing,y_spacing);
[x_interp_grid, y_interp_grid] = meshgrid(x_interp,y_interp);
plotmodes = 1:6; % Specify which modes to plot
for i = plotmodes
    modal_surf_interp =
interp2(x_spacing_grid,y_spacing_grid,modal_surfs{i},x_interp_grid,y_interp_grid);
    figure
    %surf(y_spacing,x_spacing,modal_surfs{i})
    surf(x_interp,y_interp,modal_surf_interp,'edgecolor','none')
    xlabel("Length [in]")
    ylabel("Width [in]")
    zlabel("Imaginary Component of FRF")
    title('Mode Shape at '+string(naturalfrequencies(i))+' Hz (Interpolated)')
end
```